Modelling Settling-Driven Gravitational Instabilities at the Base of Volcanic Clouds Using the Lattice Boltzmann Method

Jonathan LEMUS, Allan FRIES, Paul A. JARVIS, Costanza BONADONNA,
Bastien CHOPARD, Jonas LÄTT

02/06/2021

Introduction

- Explosive volcanic eruption ➔ large amount of particles injected in the atmosphere
- Generate multiple hazards : fine ash dispersal disturbs aviation, sedimentation affects inhabited areas…*(Blong, 2000)*
- Effective forcasting possible thanks to accurate ash dispersal models
- But: still significant differences between field observations and models *(Scollo et al. 2008)*
- ➔ Need to improve our understanding on ash sedimentation and dispersal processes

Volcanic ash in the Tubo district after December 5, 2011 eruption of Gamalama in Indonesia (Credits: AP)

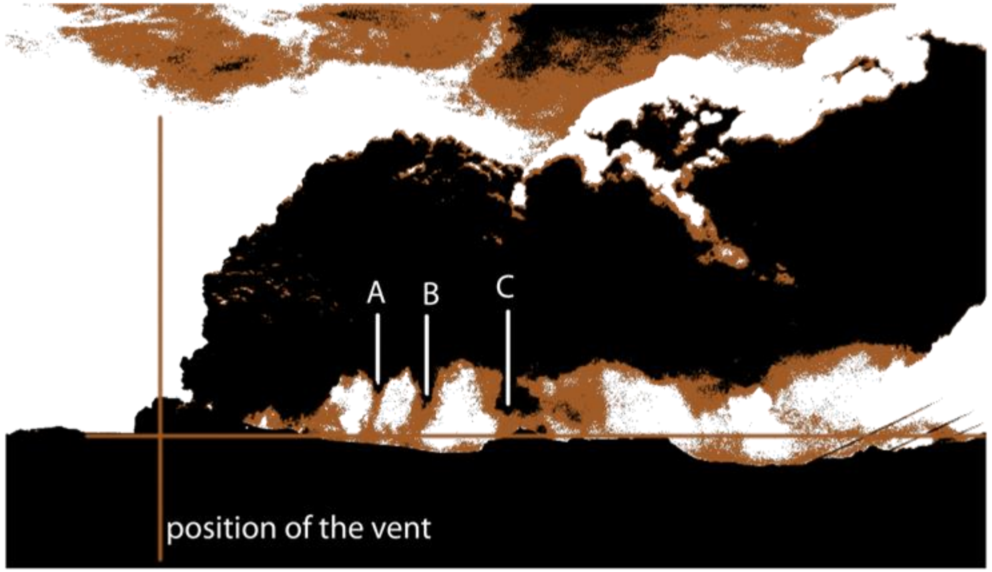
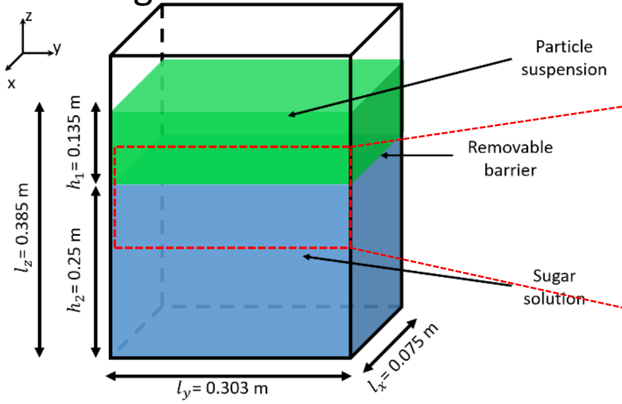


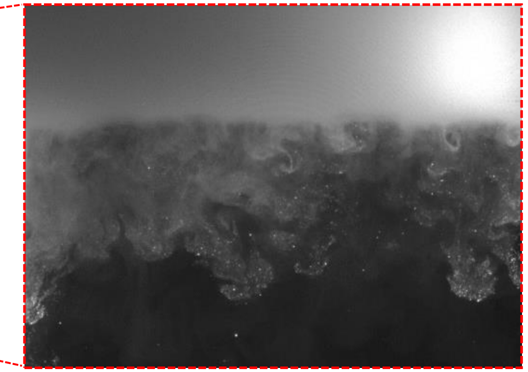
Image processing to highlight the position of downard moving fingers from the base of the volcanic cloud. Eyjafjallajökull (Iceland) 2010

Introduction

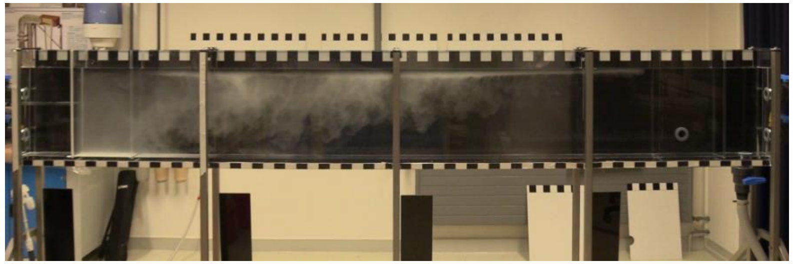
- Field investigations:
  - Image analysis with high defnition recordings
  - Study of the ground deposit
  - Particle concentration using LIDAR measurements

- Analoguous lab experiments
  - Static configuration → focus on the sedimentation process (Rayleigh-Taylor type instabilities)




Experiment snapshot (Credits: Allan FRIES)

  - Shear configuration → add the effect of shear at the interface (Rayleigh-Taylor + Kelvin-Helmholtz)


Flume experiment (Recirculating flow). (Credits: Paul JARVIS)

Introduction

- Goals:
  - ❖ Develop numerical models capable of investigating the dynamics of settling-driven gravitational instabilities (SDGIs).
  - ❖ Expand the parameter space in order to complement the observations from both experimental and field investigations.
  - ❖ Build a « general » parametrisation able to include SDGIs in general dispersal models.

- Numerical models:
  - ❖ Two-phase model simulating the individual motion of each particles.
  - ❖ Single-phase model implementing particles as a continuum phase transported by a fluid.
    - Description and motivations
    - Validation (using lab results, theory and previous investigations)
    - Additional results given by the model
    - Perspectives

Single-phase model

- Three-way coupling model between fluid momentum, fluid density and particle volume fraction.

1. Particle field described by an advection-diffusion-settling equation
2. Fluid density altered by a quantity (e.g. sugar in our experiments) described by a classical advection-diffusion equation.
3. Fluid motion described by Navier-Stokes equations

- Assumptions:
  - Particle are small enough and in large number to be considered as a continuum concentration field
  - Drag force in equilibrium with the gravitational force ➔ body force term in fluid momentum equation = buoyant force term (Boussinesq approximation)

Single-phase model

Lattice Boltzmann

$$\frac{\partial \vec{u_f}}{\partial t} + (\vec{u_f}.\vec{\nabla})\vec{u_f} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{u_f} + \vec{f}$$

$\vec{e_z}$

Fluid

$\|\vec{u_f}\|$=0

$$\vec{f} = \left[\left(\frac{\rho_p - \rho_0}{\rho_0}\right)\emptyset + \left(\frac{\rho}{\rho_0} - 1\right)(1-\emptyset)\right]\vec{g}$$

$\vec{u_f}$

$\vec{u_f}$

Sugar (density)

ρ=ρ_0

$\rho = \rho_0(1 + \alpha S)$

$$V_s = \frac{D_p^2 g[\rho_p - \rho(S)]}{18\mu}$$

Settling velocity

Particles

$\emptyset = \emptyset_0$

$\emptyset$=0

1st order Finite Difference

$$\frac{\partial \rho}{\partial t} + \vec{u_f}.\vec{\nabla}\rho = D_S\nabla^2\rho$$

3rd Order Weighted Essentially Non-Oscillatory (WENO) Finite Difference

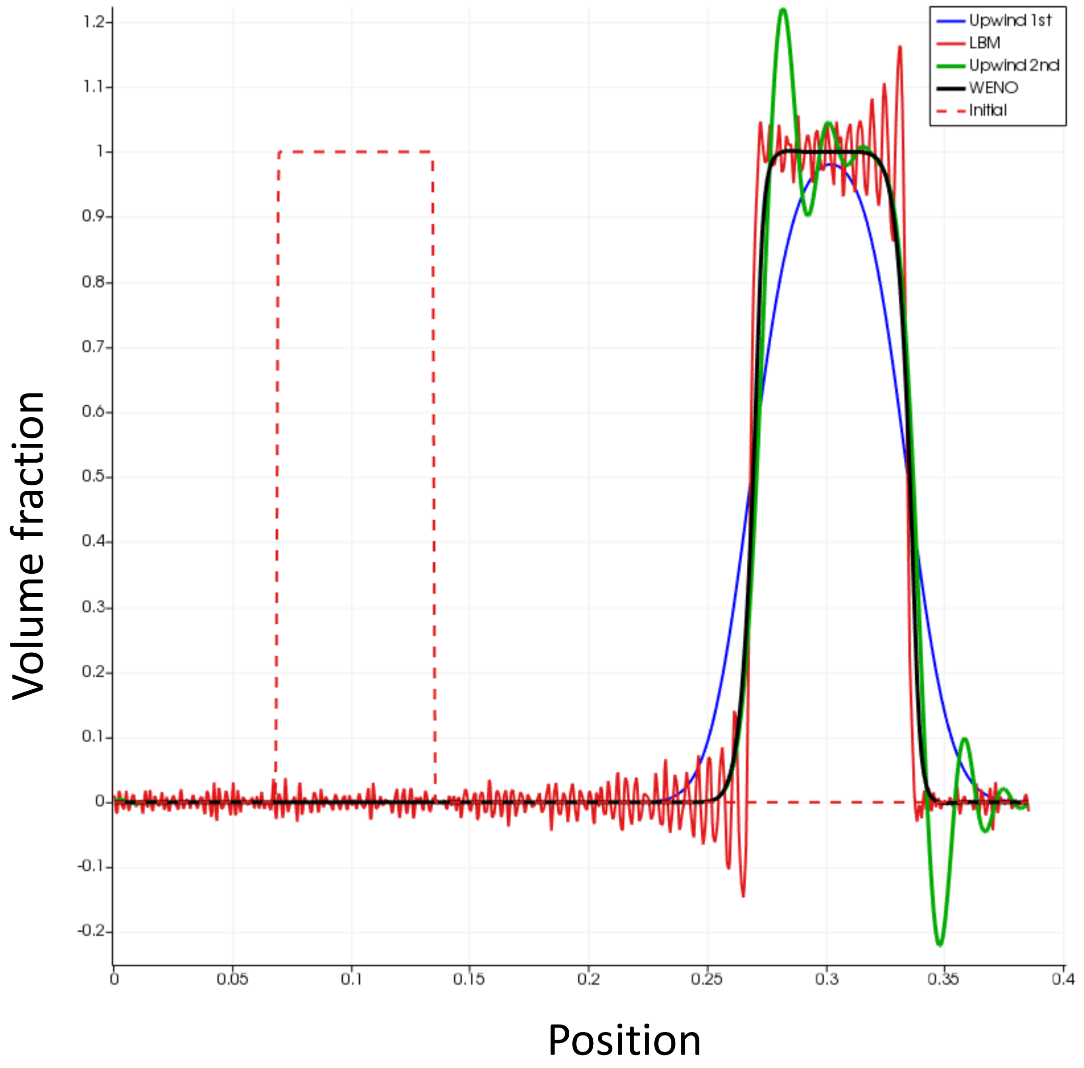
$$\frac{\partial \emptyset}{\partial t} + (\vec{u_f} - V_s\vec{e_z}).\vec{\nabla}\emptyset - \emptyset\vec{\nabla}.(V_s\vec{e_z}) = D_c\nabla^2\emptyset$$

u_f the fluid velocity
V_s the particle settling velocity
ρ_0 the liquid water density
ρ the total fluid density
ρ_p the particle density
ν the fluid kinematic viscosity
μ the fluid dynamic viscosity
$\emptyset$ the particle volume fraction
g the standard gravity
D_p the single particle diameter
S the sugar concentration
α the sugar expansion coefficient
D_c the particles diffusion coefficient
D_S the sugar diffusion coefficient

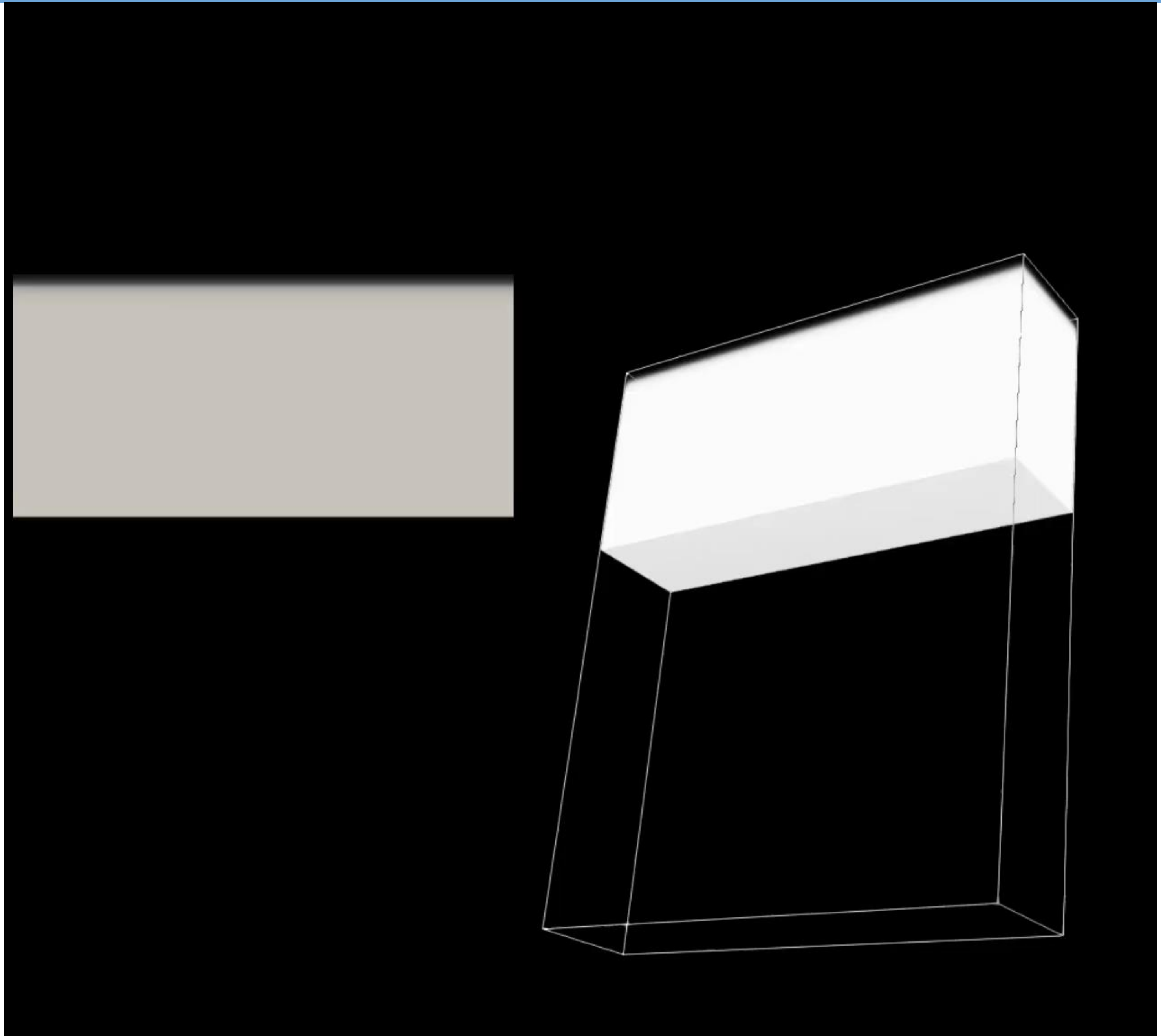
Single-phase model

- WENO (3rd order) = adaptive scheme third order accurate in smooth regions and second order near discontinuities.

- Why WENO ?
  - Easy to implement on uniform meshes → easy to couple with LBM
  - Reduced numerical diffusion compared to 1st order
  - Stable compared to LBM : in the BGK model of Advection-Diffusion, $\tau = \frac{D}{c_s^2} + \frac{\delta t}{2}$
  - No dispersion arround sharp interfaces (Total Variation Diminishing (TVD) property)



Single-phase model



Results (Validation)

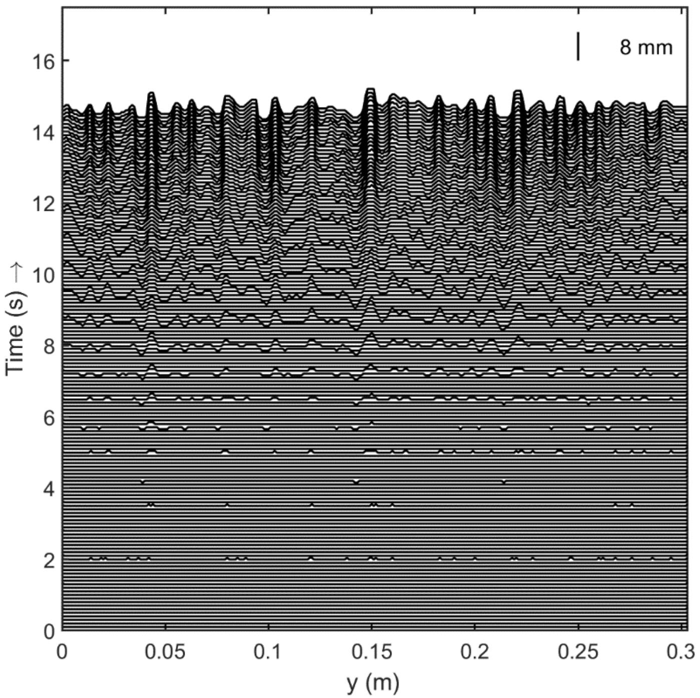
1. Early stage of the perturbation: Linear stability analysis (theory) vs. Spectral analysis of the particle interface in the simulations

2. Later stage (Non-linear): comparison with experimental investigations (Particle Boundary Layer (PBL), finger velocity…)

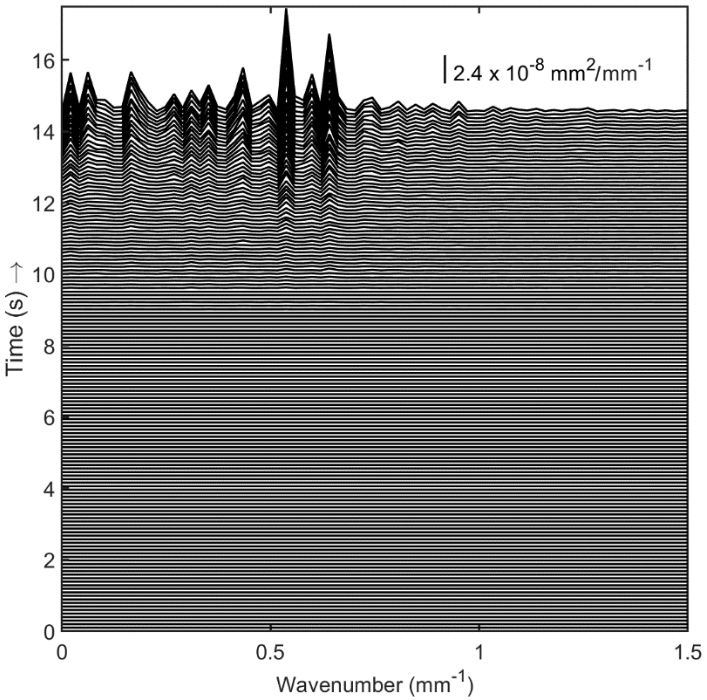
3. Comparison with extended analytical laws in the litterature (particle concentration, mass of particle deposited…)

Results - Early stage

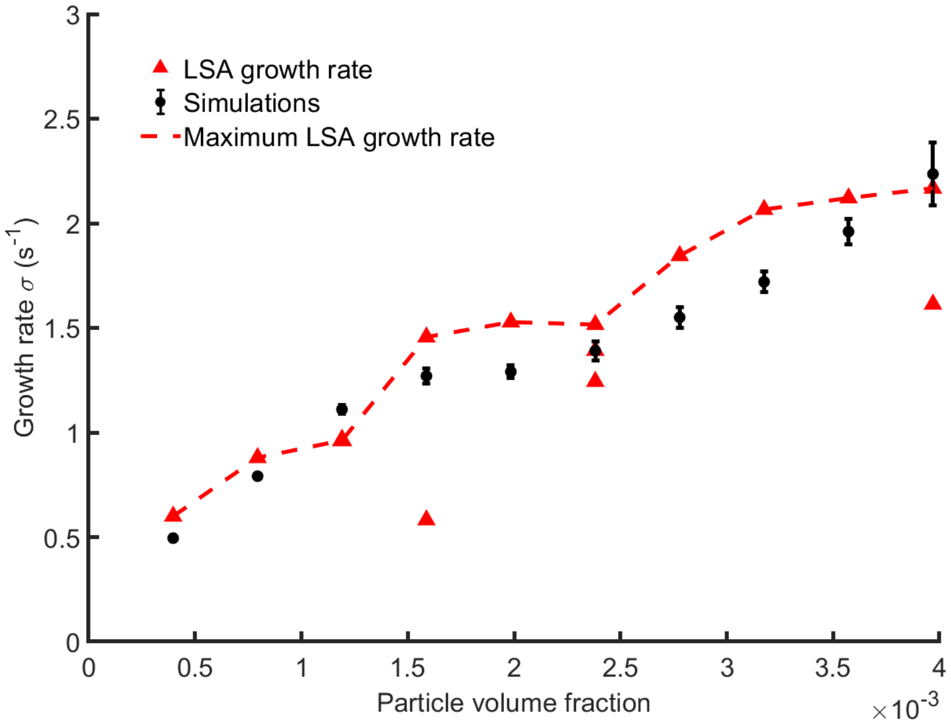
Spectral analysis of the particle interface = extract the dominant modes and their associated growth rate, assuming an exponential growth



Space time diagram of the particle interface



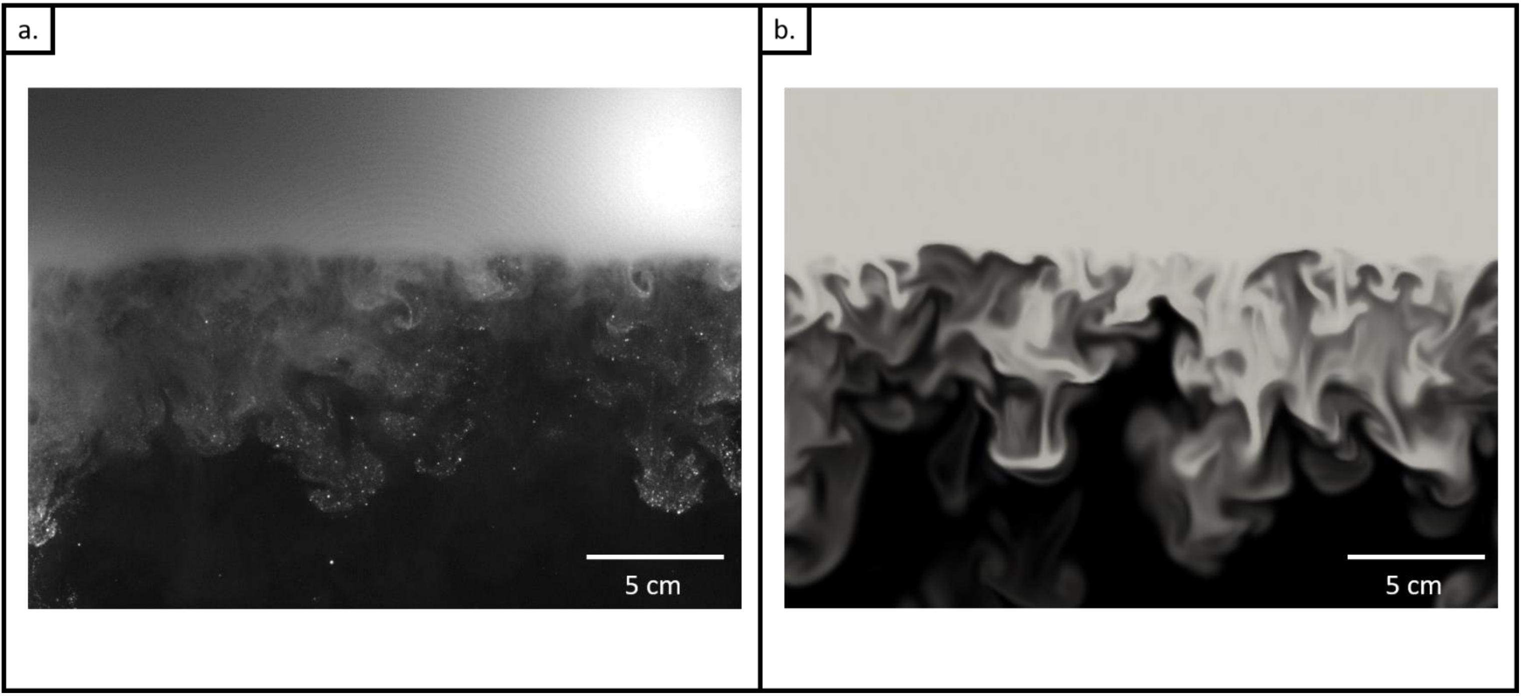
Space time diagram of the interface power spectral density



Comparison of the instability growth rate measured in the simulations (black circles) and that predicted by the linear stability analysis (red triangles)

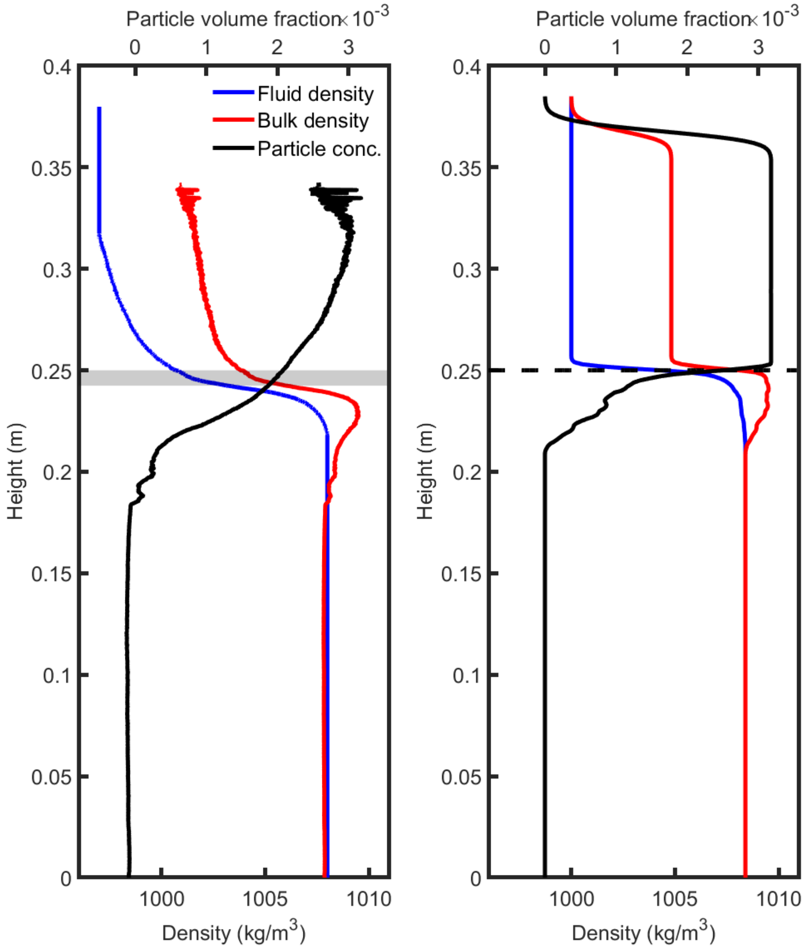
Results - Later stage

- Qualitative comparison : Similar shape of fingers especially for the eddies at the edges due to shearing.
- Similar increase of the bulk density beneath the initial interface = Particle boundary layer



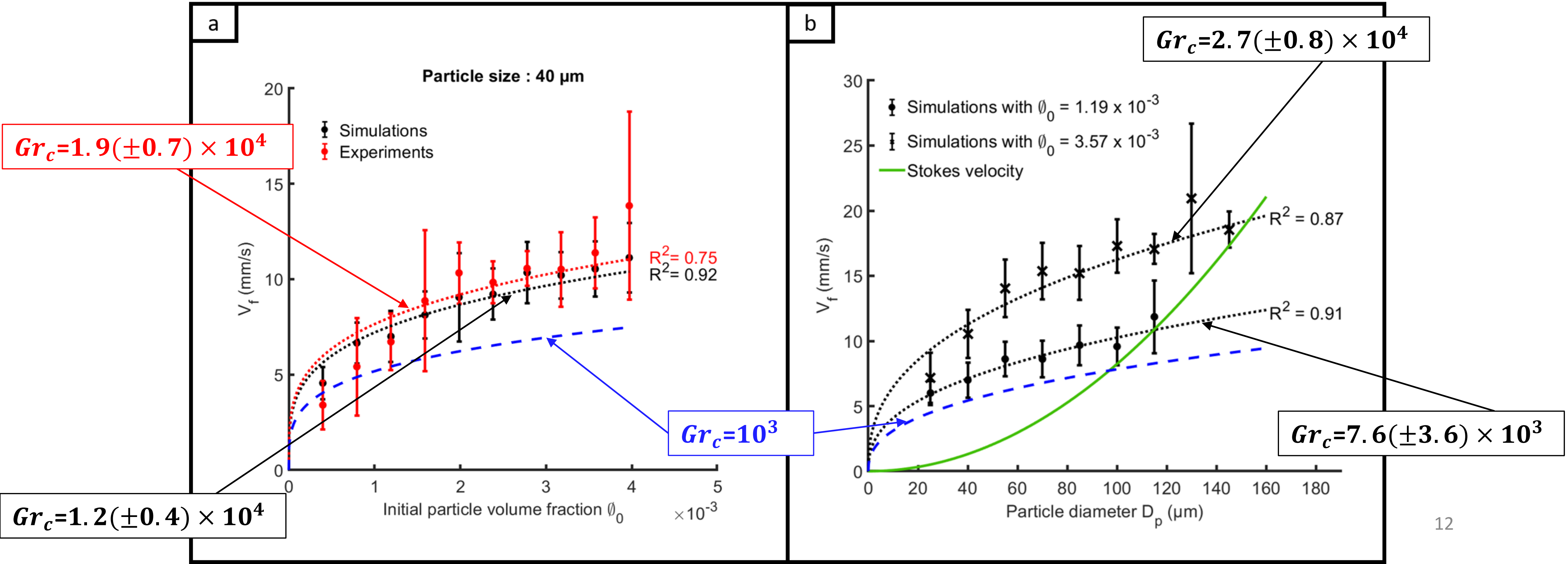
Snapshot of experiments 19.5 s after the barrier removal

Slice extracted from the 3D simulations (same moment, same configuration)



Results - Later stage

- Finger vertical velocity as function of (a) the initial particle volume fraction and (b) the particle size
- We assume $V_f = [g']^{\frac{2}{5}} \left[\frac{\pi V_S \delta_{PBL}^2}{4}\right]^{\frac{1}{5}}$, with $\delta_{PBL} = \left(\frac{Gr_c \nu^2}{g'}\right)^{\frac{1}{3}}$ (Carazzo & Jellinek, 2012; Hoyal et al., 1999)

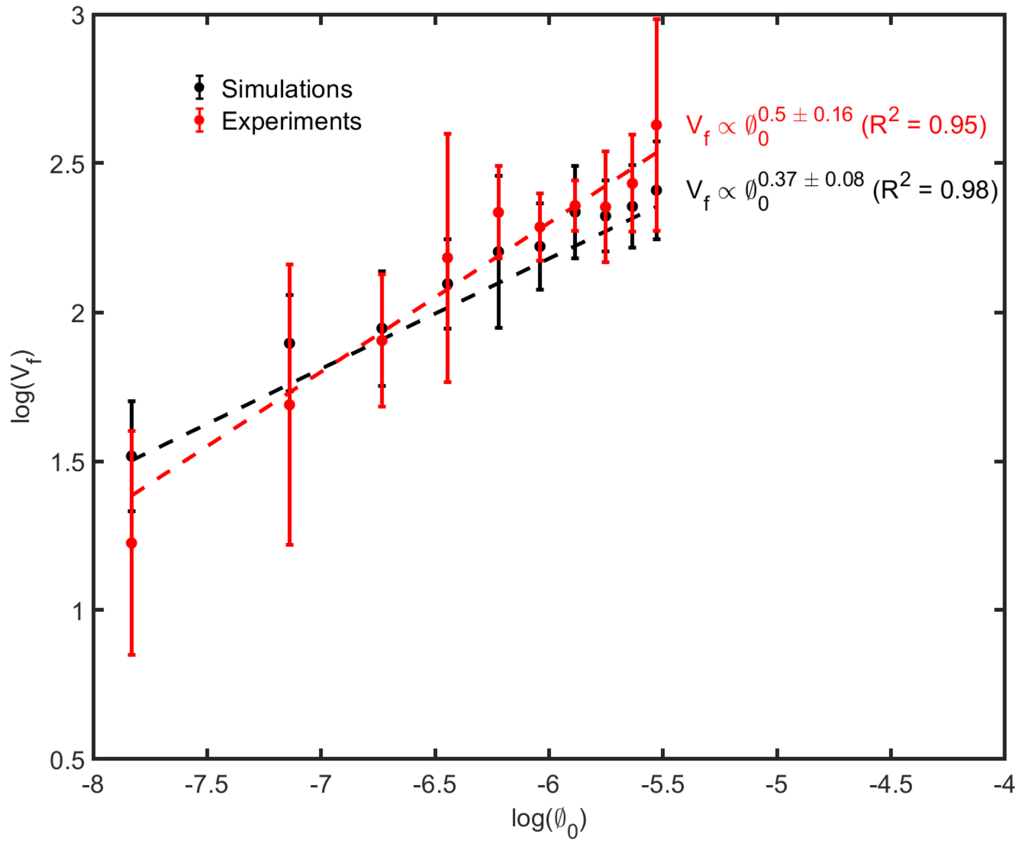


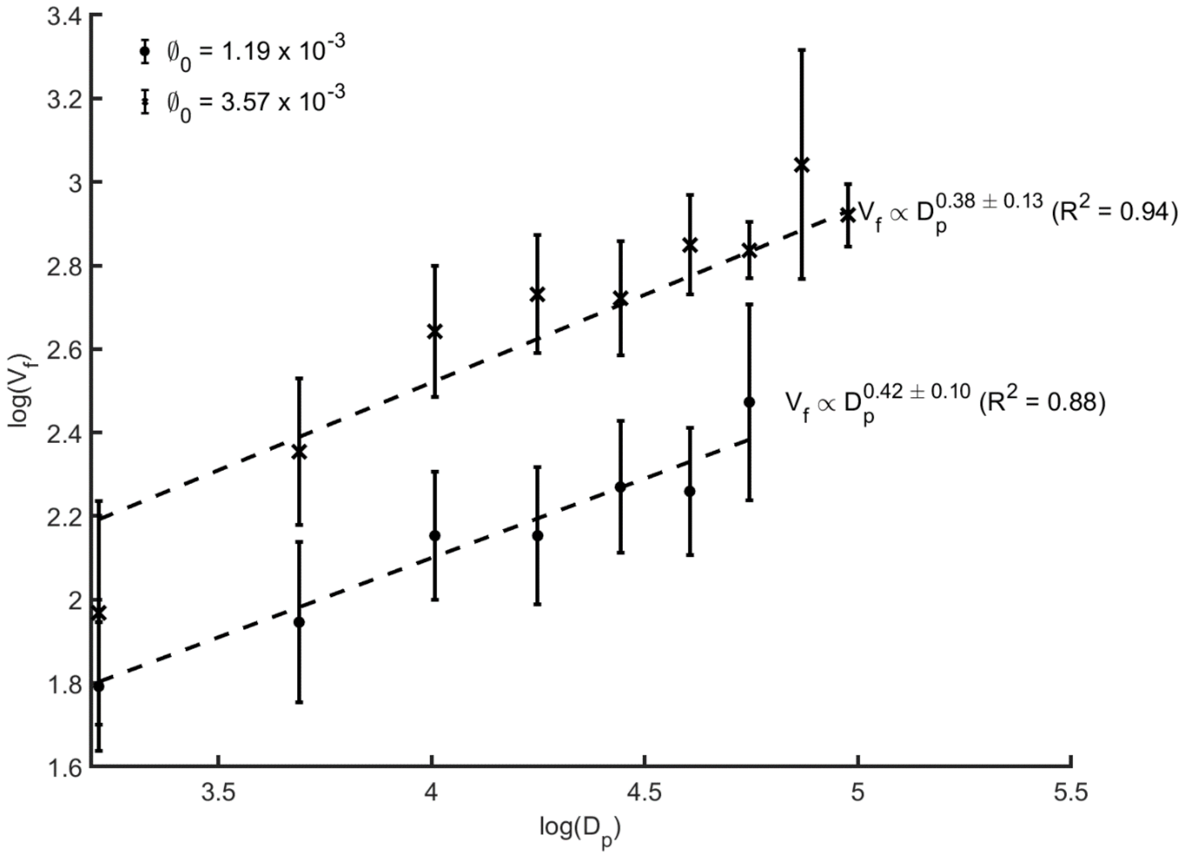
Results - Later stage

- Particle size threshold for the fingers formation:

$$D_p^* = \left[\frac{(18\mu)^2 \emptyset \delta_{PBL}}{g(\rho_p - \rho_f)\rho_f}\sqrt{\left(\frac{\pi}{4}\right)}\right]^{\frac{1}{4}}$$

Results - Later stage





$\rightarrow V_f \propto \emptyset_0^q$

The analytical formulation suggests $q = 4/15 \cong 0.27$

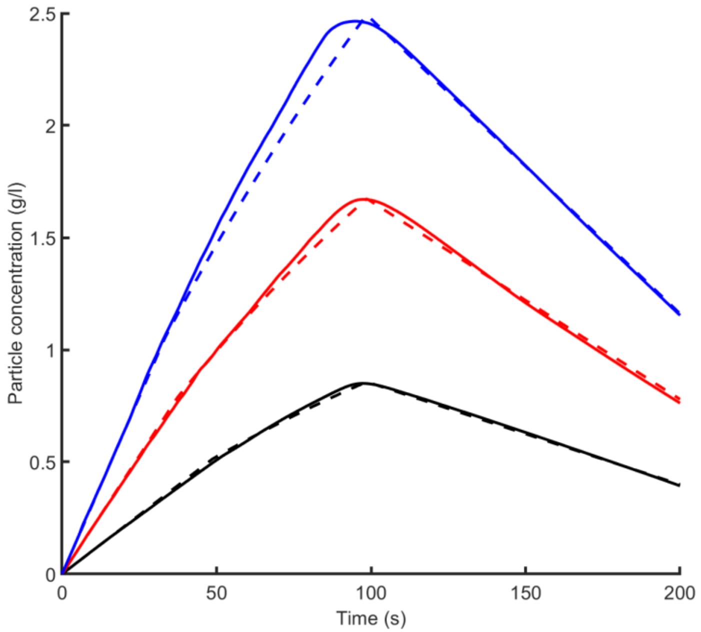
==> different formulation of δ_{PBL}?

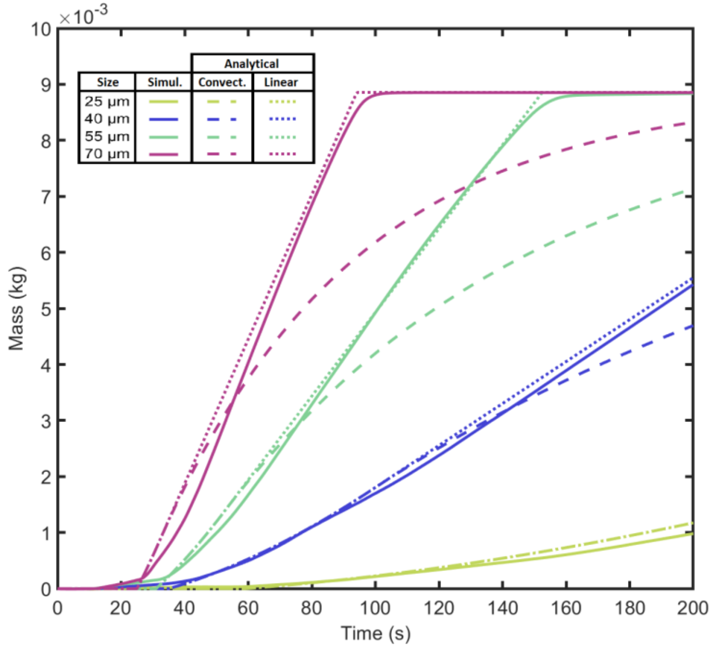
$\rightarrow V_f \propto Dp^{\eta}$

The analytical formulation suggests $\eta = 0.4$

Results - Comparison with analytical studies (Hoyal et al., 1999)

- Mass balance for different dynamics in the two layers in order to define the temporal evolution of the particle concentration and the mass of particles deposited.
- Here we focus on a [quiescent upper layer - convective lower layer].
- We extended the analysis to take in account the fact that:
    - there is a delay between the start of the experiment and the first arrival of particle at the bottom
    - after some time all particle in the upper layer have settled across the interface ==> end of convection



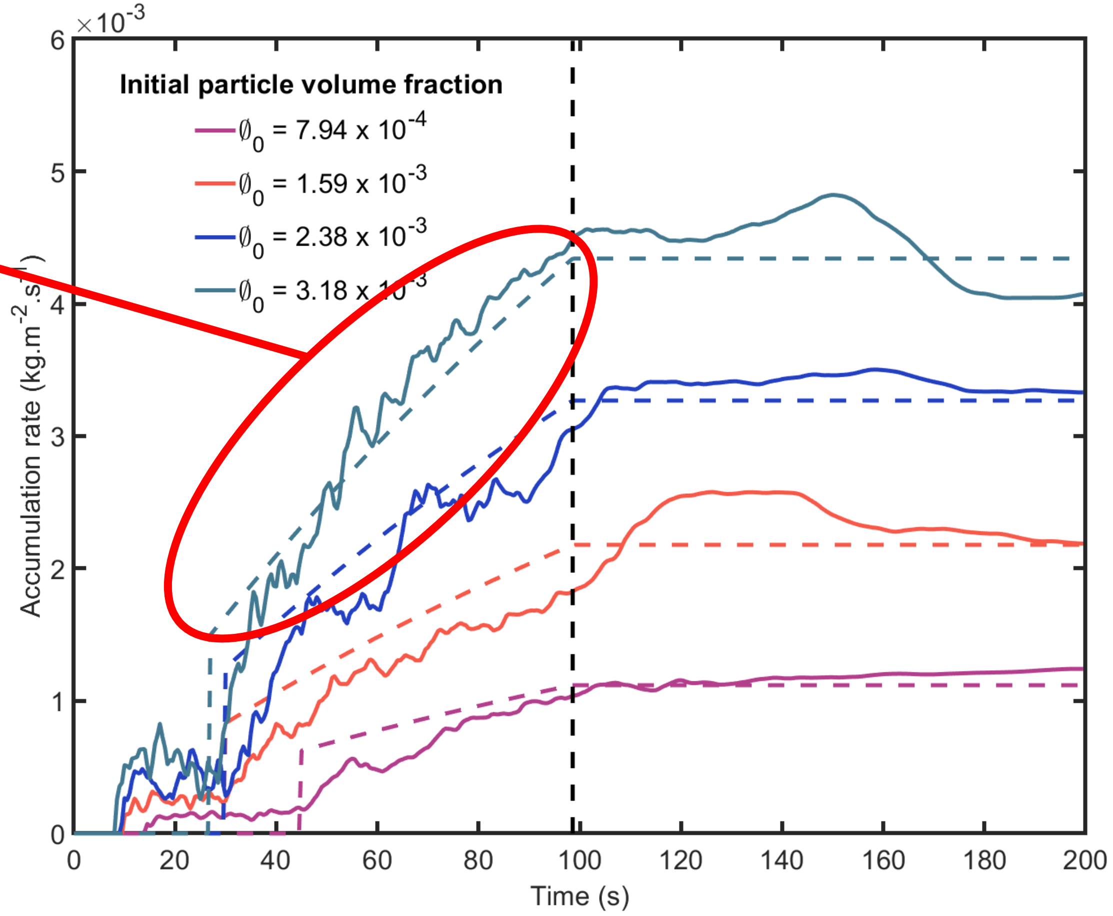


Accumulation rate

- Accumulation rate (AR) at the bottom of the tank

- Increasing AR during convective phase (i.e. during fingers downward motion). AR is supposed to be constant in the case of individual settling.

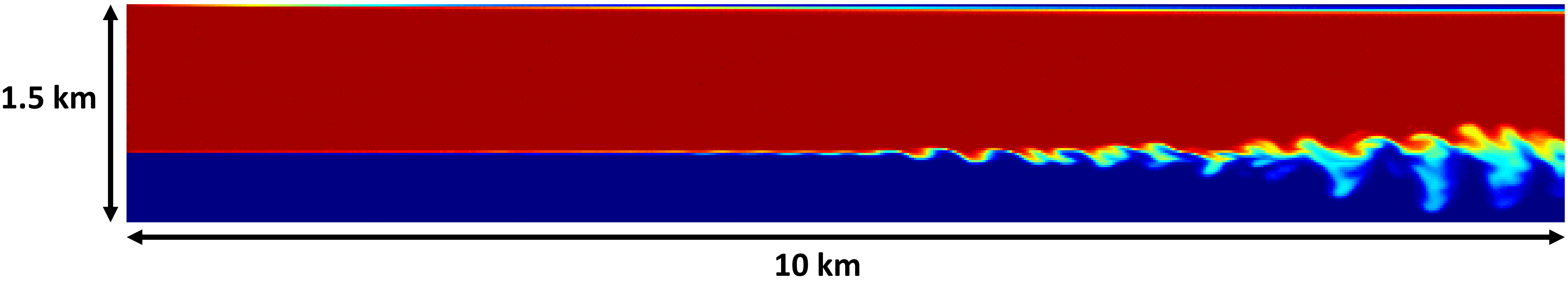
➔Specific ground signature of fingers through the accumulation rate



Accumulation rate: (solid) Simulations, (dashed) Extended analytical model. The black dashed line is the time when all particles have settled from the upper layer

Perspectives

- Crucial need for the integration of the wind to investigate on the effect of shear at the interface



Simulation in air with wind (upper: 8m/s ; lower: 11m/s), partcile size : 30µm

- Investigation on a possible coupling with the aggregation process

Conclusions

- We developed and validated the model from the linear to non-linear stage using theory, experiments and previous analytical studies.
- New insights into the value of the critical Grashof number ($Gr_c \sim 10^4$ in our configuration) ➔ Grashof number may not be the correct dimensionless form of the PBL thickness
- Need for further investigation on the PBL scaling
- The AR in the presence of fingers contrasts with the constant AR expected in individual sedimentation ➔ provides a typical signature of settling-driven gravitational instabilities on the ground

THANK YOU